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Cooperative Evasion and Pursuit for Aircraft Protection

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14. ABSTRACT
This report summarizes the work performed over a three year research effort in which wase analyzed an interception engagement in which a defending missile is red from an aircraft to intercept an incoming homing missile. Three different cooperation schemes were presented and mathematically formulated. For each case optimal cooperative guidance laws were derived, according to the constraints induced by cooperation limitations. The first case implied two-way cooperation allowing full synergy between the target and the defender. In this case no constraints were applied on their behavior which allowed us 36 of 38 to derive cooperative pursuit strategies for the target and the defender. In the second case we assumed that only one-way cooperation is available from the side of the target. As a realization of such a scenario we considered an independently homing defender and a target trying to lure in the missile. For this case the optimal one-way cooperative support strategy was derived for the target to aid the defender intercept the missile. Third approach assumed information sharing from the target to the defender, i.e. independently evading target, while the objective of the defender was to exploit this information to intercept the missile. All three guidance schemes were derived assuming arbitrary order linear dynamics of the adversaries, perfect information under the constraints of respective information sharing schemes, and a missile employing a known linear guidance strategy. Performance of the proposed guidance laws was analyzed via simulation, using the notion of Pareto fronts. As expected, it was shown that fully cooperative actions yield best performance compared to one-way cooperation schemes and one-on-one strategies. Once two-way communication cannot be established and full cooperation is not possible one player of the target-defender team must act independently. It was shown that in such a case it is still possible to intercept a homing missile using an appropriate cooperation scheme. It was also shown that different one-way cooperation schemes impose different maneuvering requirements on the respective cooperatively acting players. Results have shown that cooperatively acting defender is more effective than cooperatively acting target, since cooperative defender only needs to turn towards predicted intercept point, while cooperative target has to take into account missile reaction to its maneuver to bring the independent defender to interception point.

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Cooperative Evasion and Pursuit for Aircraft Protection

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Abstract

In this research we focused on the development of cooperative guidance strategies for a team composed of a manned or unmanned aircraft and a defending missile trying to protect the aircraft from the incoming homing interceptor. We have also looked at the problem from the point of view of the missile and have analyzed the problem as a game between two teams, the target-defender team and that of the missile.

The research has been conducted in three years. The main findings of the study in the first year appear in that year's annual report and can be summarized as follows:

- Knowing the evasive maneuver of the aircraft substantially reduces the control effort requirements from its defending missile. Choosing an appropriate target (aircraft) maneuver further reduces the defending missile control effort. The cooperation also dramatically improves the defender's homing performance, especially in scenarios where the target aircraft employs evading maneuvers.

- Fusion of measurements between the aircraft and defending missile enables fast identification of the missile's guidance strategy, making its future trajectory predictable. Moreover, in cases where the defending missile has bearing only measurements, the cooperation with the aircraft enables estimating also range and thus advanced cooperative optimal control based guidance laws can be utilized.

- Command to line of sight (CLOS) guidance for the defender missile provides superior performance over proportional navigation (PN) guidance in terms of the miss distance and the total control effort.

The main findings of the study in the second year appear in that year's annual report and can be summarized as follows:

- optimal one-on-one, non-cooperative, aircraft evasion strategies from a missile using a classical linear guidance strategy were derived. The strategies have a bang-bang structure. For realistic parameters, utilizing such optimal strategies yields a small miss distance that is not sufficient for ensuring aircraft survivability. Thus, the use of a defender missile is of essence.

- In contrast to the optimal one-on-one evasion strategy, the optimal cooperative target maneuver may be either constant or arbitrary. Implementing such a strategy the target can lure in the attacker, allowing its defender to intercept the attacking missile even in scenarios where the defenders maneuverability is at a disadvantage compared to the attacking missile, yielding hit-to-kill homing performance.

- Analytical capture envelopes were derived for scenarios where the missile and the defender use different classical guidance laws, such as PN, pure pursuit (PP), and CLOS.

In the third year of the research we first looked at the problem as a game between two teams (the target-defender team and that of the missile). Then we looked into the effect of different cooperation/communication schemes on the performance of the target-defender team. The main findings of the study in the third year of the research are:

- For a given missile strategy (e.g. PN and PP) the mission outcome relies heavily on the defender guidance strategy (e.g. PN and CLOS guidance), and vice versa. Thus, for a realistic implementation of a guidance law, from a given set of laws, a game theoretic analysis is required. Formalizing the engagement as a two-person zero sum game, optimal strategies are obtained not only in pure strategies but also in mixed strategies (i.e. a probabilistic distribution over the set of pure strategies).

- Three different cooperation schemes were investigated: 1) two-way cooperation where the target-defender team employs its optimal cooperative strategy, 2) one-way cooperation realized by defender employing a classical one-on-one guidance law while the target helps it by luring in the missile, 3) one-way cooperation realized by target employing an arbitrary evasive strategy while the defender attempts to reach the predicted interception point. Using the notion of Pareto fronts it is shown that the performance of the target and the defender is highly dependent on the cooperation scheme, where as expected the two-way cooperation scheme provides the best performance. Yet, one-way cooperation schemes attain considerably better results than independent actions.

- Once two-way communication cannot be established and full cooperation is not possible one player of the target-defender team must act independently. In such a case it is still possible to intercept a homing missile using an appropriate cooperation scheme. Different one-way cooperation schemes impose different maneuvering requirements on the respective cooperatively acting players. Results have shown that cooperatively acting defender is more effective than cooperatively acting target, since cooperative defender only needs to turn towards predicted intercept point, while cooperative target has to take into account missile reaction to its maneuver to bring the independent defender to interception point.

The results of the research have appeared in 3 published and 1 accepted journal publications, see:

- Shaferman, V. and Shima, T., "Cooperative Multiple Model Adaptive Guidance for an Aircraft Defending Missile", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 33, No. 6, 2010, pp. 1801-1813, DOI: 10.2514/1.49515.
- Shima, T., "Optimal Cooperative Pursuit and Evasion Strategies Against a Homing Missile", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 2, 2011, pp. 414-425, DOI: 10.2514/1.51765
- Ratnoo, A. and Shima, T. "Line-of-Sight Interceptor Guidance for Defending an Aircraft", *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 2, 2011, pp. 522-532, DOI: 10.2514/1.50572
- Ratnoo, A. and Shima, T., "Guidance Laws Against a Defender-Evader Team", *AIAA Journal of Guidance, Control, and Dynamics*, to appear in 2012.

The last part of the research, presented in depth in this report, has been accepted for conference presentation:

- Prokopov, O. and Shima, T., "Linear Quadratic Optimal Cooperative Strategies for Active Aircraft Protection", *AIAA Guidance, Navigation, and Control Conference*, Minneapolis, Minnesota, August, 2012.

and has also been recently submitted to the *AIAA Journal of Guidance, Control, and Dynamics.*

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I. Introduction

Traditional interception engagements are concerned with a single missile chasing a single target. The classical and most widely used approach in the solution of such guidance problems assumes perfect information, linearized kinematics,[1] and applies optimal control theory tools. The well-known proportional navigation (PN) guidance law[2] represents the optimal solution for the simplest scenario where interceptor has ideal dynamics and the target does not maneuver. Augmented proportional navigation (APN)[3] represents expanded optimal solution for the scenario where a target performs a known constant maneuver. Further extension of APN is an optimal guidance law (OGL)[4] where an additional assumption is that the interceptor has first order dynamics. In Ref. 5 two one-on-one guidance laws were proposed for imposing a terminal interception angle. These strategies were derived using linear quadratic optimal control and differential games approaches for arbitrary order linear missile dynamics.

In order to answer the threat coming from homing missiles employing such guidance laws, a significant effort was made on extending protection capabilities of targeted aircrafts. Among the systems developed for this purpose are electronic countermeasures and various kinds of decoys. Besides these means of protection an aircraft can perform an evasive maneuver, which can be either arbitrary or optimally adjusted against the incoming interceptor. It is possible to develop an optimal evasion strategy using optimal control theory tools, however it requires information on future pursuer's behavior, i.e. it's guidance law. A case study where such a problem was formulated as a one-sided optimal control problem against a PN guided interceptor was presented in Refs. 6–8. In these works some simplifying assumptions were applied, such as two dimensional analysis and constant pursuer and evader speeds with bounded maneuverabilities. A non-linear engagement dynamics along with assumptions on first order pursuer dynamics was considered for problem formulation in Ref. 6. A numerical solution was presented over a set of various engagement's initial conditions. In Ref. 7 a linearization around the collision course was made assuming ideal pursuer dynamics. This study was extended in Ref. 8 to a non-linear model. The resulting optimal evasion strategies obtained in the above works were found to have a bang-bang structure, i.e. applying maximum available acceleration normal to the line-of-sight for a given period of time.

Continued development of sophisticated interceptor missiles having a large maneuver capability while employing advanced guidance laws render the application of evasive maneuvers insufficient. To increase the probability of survival the target aircraft may also deploy a defender missile to intercept an incoming threat. Such a multi-agent scenario is denoted as target-missile-defender engagement and it may be treated for example by using either differential games or optimal theory tools. In Ref. 9 such a scenario was presented as a two team three person dynamic game - the Lady, the Bandit and the Bodyguard. The bandit's (i.e. missile) objective is to capture the lady (i.e. target), while lady and her body-guard's (i.e. defender) objective is to prevent it. The body-guard is trying to intercept the bandit prior to its arrival within the lady's proximity. In a recent paper[10] this game was reformulated as a target-missile-defender problem. Analytic terms were derived for the ideal dynamics case and the solution provided the cooperative strategies for the aircraft and defending missile, as well as the optimal combined pursuit-evasion strategy for the attacking missile. Another recent work,[11] performed in the first year of this research, investigated the target-missile-defender interception problem for a case where the target and its defender share noisy measurements of the attacking missile. A nonlinear adaptation of a multiple model adaptive estimator was used in order to identify the guidance law and the guidance parameters of the incoming homing missile. A matched defender's missile guidance law was optimized to the identified homing missile guidance law and the target's guidance law was set to minimize the control effort of the defender.

In a recent work,[12] performed during the second year of this research, the target-missile-defender engagement, where the missile is driven by a linear guidance law such as PN, APN, and OGL was analyzed and cooperative pursuit and evasion strategies for the defender and the target were derived. The derivation was based on arbitrary-order linear adversaries dynamics and perfect information. The optimal noncooperative one-on-one evasion strategies from a missile employing a linear guidance strategy were also analytically derived. It was shown that a hit-to-kill performance is achievable due to cooperation, even when the defending missile has a considerable maneuverability disadvantage over the interceptor.

Another implementation of the cooperation concept between the target and the defender in target-missile-defender engagement was presented in Ref. 13 by using the line-of-sight (LOS) guidance scheme. In this work (performed during the first year of this research) LOS guidance kinematics with a maneuvering launch platform (defended aircraft) was derived and investigated. A cooperative guidance law was proposed for the defended aircraft based on the kinematic results to maximize the attacker-to-defender lateral acceleration ratio. The proposed cooperative guidance scheme was studied analytically and via simulations for various attack geometries showing better relative control effort performance.

In a recent work[14] (performed during the second and third years of this research, and presented also in this year's interim report) an analysis of target-missile-defender engagement was carried out where the defender employs proportional navigation and LOS guidance against proportional navigation and pure pursuit missile strategies, while the target was assumed to follow a non-maneuvering flight. Closed form expressions for lateral acceleration ratios and capture zones were presented as well as an analytic expression for attacking missile initial position and launch angles for a successful evasion from the defender.

In the final stage of this research, presented next, we investigated the effect of cooperation on the target-missile-defender engagement. We consider three approaches for cooperative defense of an aircraft from a homing missile, using a defender missile. First, we consider a case where the target and the defender are able to maintain a steady communication link allowing two-way cooperation. In such a formulation, full cooperation is available between the target and the defender. In the second case we assume that only one-way communication is available to the defender and it is unaware of the target (aircraft) actions, i.e. the defender transmits the required data to the target and acts independently. For this case a cooperative strategy is derived for the target to aid a defender which is employing a classical one-on-one guidance law to hit the attacking missile. For the third case one-way communication is assumed for the target which shares its evasive maneuver with the defender. Thus, we derive a cooperative pursuit strategy for the defender while the target performs an arbitrary evasive maneuver, that is known to the defender. We assume perfect information in all three cases except described information sharing limitations and the fact that the missile is unaware of the existence of the defender. All cases are based on linearized kinematics, and adversaries are assumed to have arbitrary order linear dynamics.

The remainder of this report is organized as follows. In the next section the target-missile-defender engagement is formulated and a respective mathematical model is provided. Then, a generalized form of linear guidance laws is presented including review of classical special cases of PN, APN, and OGL. Next, the cooperative pursuit and evasion strategies are derived for the defender and the target. This is followed by the derivation of a cooperative strategy for a target with independently guided defender. Derivation of cooperative defender strategy with independently evading target is presented next. Then, a simulation analysis is presented, followed by concluding remarks.

II. Problem Formulation

We consider an engagement scenario consisting of three entities: an attacking missile (M), an evading aircraft (T) and a defender missile (D). A defender missile is launched by evading aircraft in order to intercept the incoming threat. The attacking missile is unaware of the defender and employs a known linear one-on-one guidance law to catch the target.

For derivation purposes the missile-target-defender scenario is assumed to take place in a plane. Three entities form two collision triangles. First, between the target aircraft and the attacking missile and second between the defender missile and the attacking missile. We assume that the engagement occurs in the endgame phase, where deviations from the respective collision triangles are small and therefore the linearization along the initial lines of sight (LOS_0) is justified. Adversaries speeds are assumed to remain constant during the endgame. In addition, for simplicity, we assume that the defender is launched from the targeted aircraft and therefore we can unite the initial line of sights of missile-target and defender-missile pairs.

A schematic view of the planar end-game engagement geometry is shown in Fig. 1. The X axis is aligned with the initial line of sight (LOS_0) used for the linearization and Y is perpendicular to it. The range between the missile and target is denoted r_{MT}, while that between the defender missile and the attacking one is denoted r_{MD}. We denote y_{MT} and y_{MD} as the target-missile and missile-defender relative displacements normal to LOS_0, respectively. The missile's, target's, and defender's accelerations perpendicular to LOS_0 are denoted by a_M, a_T, and a_D, respectively.

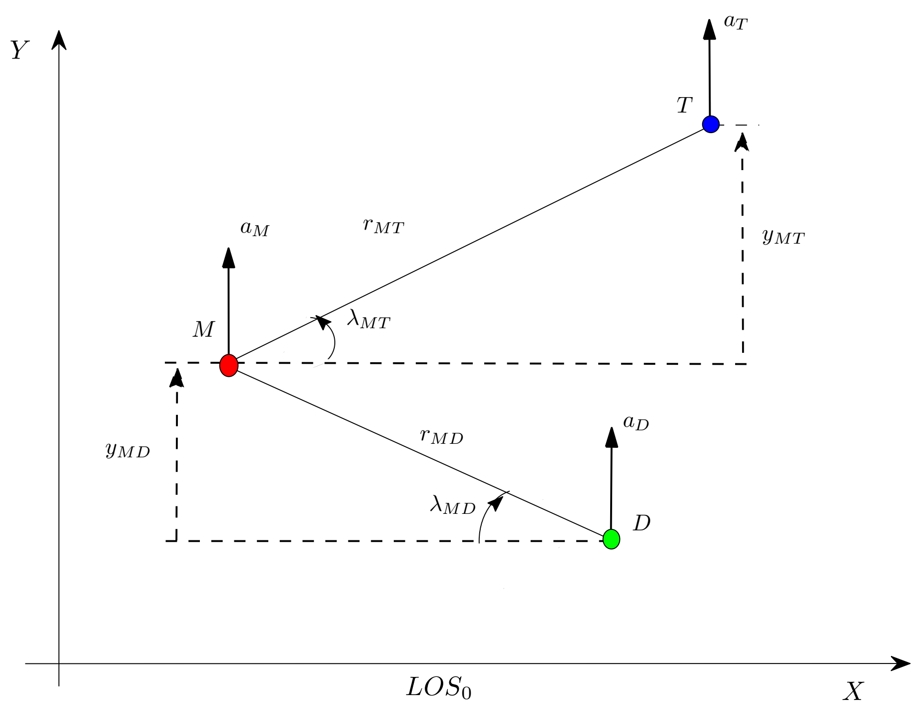

Figure 1. Target-missile-defender engagement geometry.

A. Equations of Motion

We assume that during the endgame adversaries dynamics can be represented by arbitrary order linear equations

$$\dot{\boldsymbol{x}}_i = \mathbf{A}_i \boldsymbol{x}_i + \mathbf{B}_i u_i \quad ; \quad i = \{M, T, D\} \tag{1}$$

$$a_i = \mathbf{C}_i \boldsymbol{x}_i + d_i u_i \quad ; \quad i = \{M, T, D\} \tag{2}$$

where $\boldsymbol{x}_i$ is the state vector of an agent's internal state variables with $\dim(\boldsymbol{x}_i) = n_i$ and u_i is its controller. The first term on the right-hand side of Eq. (2) is the part of the acceleration with dynamics and we denote it as a_{iS} (i.e. $a_{iS} = \mathbf{C}_i \boldsymbol{x}_i$). For example, if the missile has ideal dynamics ($a_{iS} = 0$) then the direct lift is obtained immediately, i.e. $a_i = d_i u_i$ therefore satisfying $\mathbf{A}_i = \mathbf{B}_i = \mathbf{C}_i = 0$. On the other hand, a missile with first order strictly proper dynamics with time constant τ_i is represented by $\mathbf{A}_i = -1/\tau_i$, $\mathbf{B}_i = 1/\tau_i$, $\mathbf{C}_i = 1$ and $d_i = 0$.

The state vector of the linearized interception problem is therefore defined as

$$\boldsymbol{x} = \begin{bmatrix} y_{MT} & \dot{y}_{MT} & \boldsymbol{x}_M^T & \boldsymbol{x}_T^T & y_{MD} & \dot{y}_{MD} & \boldsymbol{x}_D^T \end{bmatrix}^T \tag{3}$$

Defining the state vector of the linearized missile-target engagement as

$$\boldsymbol{x}_{MT} = \begin{bmatrix} y_{MT} & \dot{y}_{MT} & \boldsymbol{x}_M^T & \boldsymbol{x}_T^T \end{bmatrix}^T \tag{4}$$

and that of the missile-defender as

$$\boldsymbol{x}_{MD} = \begin{bmatrix} y_{MD} & \dot{y}_{MD} & \boldsymbol{x}_D^T \end{bmatrix}^T \tag{5}$$

we can rewrite it in a short form

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_{MT}^T & \boldsymbol{x}_{MD}^T \end{bmatrix}^T \tag{6}$$

and $dim(\boldsymbol{x}) = n_M + n_T + n_D + 4$.

The states x_1 and $x_{n_M+n_T+3}$ are the differences between the target and missile positions and between the missile and defender positions normal to the initial line of sight. x_2 and $x_{n_M+n_T+4}$ are therefore the relative respective lateral speeds, and their derivatives are the relative lateral accelerations of missile-target and missile-defender duos. Thus, the equations of motion (EOM) which represent the engagement's kinematics and dynamics can be written as

$$\dot{\boldsymbol{x}} = \begin{cases} \dot{x}_1 = & x_2 \\ \dot{x}_2 = & a_T - a_M \\ \dot{\boldsymbol{x}}_M = & \mathbf{A}_M \boldsymbol{x}_M + \mathbf{B}_M u_M \\ \dot{\boldsymbol{x}}_T = & \mathbf{A}_T \boldsymbol{x}_T + \mathbf{B}_T u_T \\ \dot{x}_{n_M+n_T+3} = & x_{n_M+n_T+4} \\ \dot{x}_{n_M+n_T+4} = & a_M - a_D \\ \dot{\boldsymbol{x}}_D = & \mathbf{A}_D \boldsymbol{x}_D + \mathbf{B}_D u_D \end{cases} \tag{7}$$

These equations can be written in vector form as

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}[u_T \; u_D]^T + \mathbf{C}u_M \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{MT} & [0] \\ \mathbf{A}_{21} & \mathbf{A}_{MD} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_{MT} & [0] \\ [0] & \mathbf{B}_{MD} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{MT} \\ \mathbf{C}_{MD} \end{bmatrix} \tag{9}$$

and

$$\mathbf{A}_{MT} = \begin{bmatrix} 0 & 1 & [0] & [0] \\ 0 & 0 & -\mathbf{C}_M & \mathbf{C}_T \\ [0] & [0] & \mathbf{A_M} & [0] \\ [0] & [0] & [0] & \mathbf{A_T} \end{bmatrix}, \quad \mathbf{B}_{MT} = \begin{bmatrix} 0 \\ d_T \\ [0] \\ \mathbf{B}_T \end{bmatrix}, \quad \mathbf{C}_{MT} = \begin{bmatrix} 0 \\ -d_M \\ \mathbf{B}_M \\ [0] \end{bmatrix} \tag{10}$$

$$\mathbf{A}_{21} = \begin{bmatrix} [0] & [0] & [0] \\ [0] & \mathbf{C}_M & [0] \\ [0] & [0] & [0] \end{bmatrix}, \mathbf{A}_{MD} = \begin{bmatrix} 0 & 1 & [0] \\ 0 & 0 & -\mathbf{C}_D \\ [0] & [0] & \mathbf{A}_D \end{bmatrix}, \mathbf{B}_{MD} = \begin{bmatrix} 0 \\ -d_D \\ \mathbf{B}_D \end{bmatrix}, \mathbf{C}_{MD} = \begin{bmatrix} 0 \\ d_M \\ [0] \end{bmatrix} \tag{11}$$

with [0] denoting a matrix of zeros with appropriate dimensions.

B. Timeline

The initial range between the attacking missile and the target is $r_{MT}(0)$. Similarly, between the attacking missile and the defender it is $r_{MD}(0)$. Under the linearization assumption around initial collision triangle, closing speeds of missile-target $V_{C_{MT}}$ and missile-defender $V_{C_{MD}}$ are assumed to be constant. Thus, the interception time is fixed and can be approximated by

$$t_{f_{MT}} = r_{_{MT}}(0)/V_{C_{MT}} \tag{12}$$

and similarly

$$t_{f_{MD}} = r_{_{MD}}(0)/V_{C_{MD}} \tag{13}$$

We define Δt as the time difference between interceptions

$$\Delta t = t_{f_{MT}} - t_{f_{MD}} \tag{14}$$

To fulfil its task the defending missile has to reach the attacking missile prior to the missile reaching the target. Thus, we require that the missile-defender engagement terminates prior to that of missile-target, i.e. $\Delta t > 0$ ($t_{fMD} < t_{fMT}$).

We define the times-to-go of the missile-target ($t_{go_{MT}}$) and missile-defender ($t_{go_{MD}}$) engagements as follows

$$
\begin{aligned}
t_{go_{MT}} &= t_{f_{MT}} - t \\
t_{go_{MD}} &= t_{f_{MD}} - t
\end{aligned}
\tag{15}
$$

In the scope of our work we are interested in the part of the engagement until the termination of the defender. Therefore, we denote the time-to-go of the engagement as

$$
t_{go} = t_{go_{MD}} \tag{16}
$$

Using the time-to-go definition above and Eq. (14) we can obtain

$$
t_{go_{MT}} = t_{go} + \Delta t \tag{17}
$$

C. Missile Guidance Law

A considerable effort was made over the years in development of various guidance laws. Assumptions as perfect information, linear kinematics, and unbounded controls were generally used for the derivations. This common practice resulted in a wide variety of guidance laws which all have the same linear form as a function of missile-target engagement state variables and possibly the target's control

$$
u_M = \mathbf{K}(t_{go_{MT}})\boldsymbol{x}_{MT} + K_u(t_{go_{MT}})u_T \tag{18}
$$

where

$$
\mathbf{K}\left(t_{go_{MT}}\right) = \begin{bmatrix} K_1 & K_2 & \mathbf{K}_M & \mathbf{K}_T \end{bmatrix} \tag{19}
$$

Among the family of linear guidance laws are the well-known classical guidance laws of PN, APN, and OGL. These guidance laws are more widely known in the following form[1]

$$
u_M = N_j' \frac{Z_j}{t_{go_{MT}}^2} \quad ; \quad j = \{PN, APN, OGL\} \tag{20}
$$

where Z_j is the zerro-effort-miss distance (ZEM) associated with each guidance law. Zero-effort-miss is defined based on the homogeneous solution of the linear dynamics model of a given engagement. In general it represents the miss distance if from the current time and until the end of the engagement no further acceleration commands are issued by the pursuer, and the target follows an assumed maneuvering model. For the PN, APN, and OGL it is given by

$$Z_{PN} = y_{MT} + \dot{y}_{MT} t_{go_{MT}} \tag{21}$$

$$Z_{APN} = Z_{PN} + a_T t_{go_{MT}}^2 / 2 \tag{22}$$

$$Z_{OGL} = Z_{APN} - a_M \tau_M^2 \psi(t_{go_{MT}} / \tau_M) \tag{23}$$

where τ_M is the missile's acceleration dynamics time constant and

$$\psi(\xi) = \exp(-\xi) + \xi - 1 \tag{24}$$

$N^{'}$ is the respective effective navigation gain, where

$$N^{'}_{PN} = 3 \tag{25}$$

$$N^{'}_{APN} = 3 \tag{26}$$

$$N^{'}_{OGL} = \frac{6\theta_{MT}^2 \psi(\theta_{MT})}{3 + 6\theta_{MT} - 6\theta_{MT}^2 + 2\theta_{MT}^3 - 3e^{-2\theta_{MT}} - 12\theta_{MT} e^{-\theta_{MT}}} \tag{27}$$

and θ_{MT} is the normalized time-to-go in the missile-target engagement

$$\theta_{MT} = t_{go_{MT}} / \tau_M \tag{28}$$

Because linear guidance laws are widely used in modern interceptors and therefore are most likely to be encountered, we assume in our further derivations that the attacking missile employs such a linear guidance strategy.

III. Two-Way Cooperative Pursuit Strategies

We first consider the general case where the target and the defender are acting as a team, with the common goal of intercepting the incoming missile by the defender. For this purpose two-way communication must be established between the target and the defender to allow telemetric data sharing that is needed for cooperative action. For example, an unmanned aerial vehicle (UAV) may fire a defender missile as countermeasure against an interceptor missile. We assume that interceptor missile guidance law is identified, i.e. its behavior and reaction to maneuvers issued by the UAV are known. Thus, the UAV may transmit it's future maneuvering sequence to the defender to allow it to predict the intercepting point with the missile and head towards it. The resulting upcoming defender's maneuver is then transmitted back to the UAV and it may issue a correction command in order to assist the defender. The UAV will actually act like a bait luring in the missile, while the defender will head towards the predicted intercept point. As a result, the missile will be intercepted with minimum target's and defender's control efforts. In this section we derive such optimal cooperative pursuit strategy for the defender and for the target against a homing missile. The underlying assumptions are that the defender and the target share perfect information on their current states and on the controls they are going to issue. It is also assumed that the missile's guidance strategy is known to the target and the defender, as it may be identified using a technique such as that proposed in the first year of the research.[11] For the reasons presented in the previous section we also assume that the missile is guided by a linear guidance strategy.

A. Cooperative Pursuit Dynamics

The linearized EOM of missile-target-defender engagement was presented in Eq. (8). By substituting the missile guidance law from Eq. (18) into Eq. (8) we obtain the EOM of the cooperative, two controllers problem

$$\dot{\boldsymbol{x}} = \mathbf{A}_{PE}(t_{go}, \Delta t)\boldsymbol{x} + \mathbf{B}_{T_{PE}} u_T + \mathbf{B}_{D_{PE}} u_D \tag{29}$$

where

$$\mathbf{A}_{PE}(t_{go}, \Delta t) = \begin{bmatrix} \mathbf{A}_{MT_{PE}}(t_{go}+\Delta t) & [0] \\ \mathbf{A}_{21_{PE}}(t_{go}, \Delta t) & \mathbf{A}_{MD} \end{bmatrix} \tag{30}$$

and

$$\mathbf{A}_{MT_{PE}}(t_{go}+\Delta t) = \begin{bmatrix} 0 & 1 & [0] & [0] \\ -d_M K_1 & -d_M K_2 & -(\mathbf{C}_M + d_M \mathbf{K}_M) & \mathbf{C}_T - d_M \mathbf{K}_T \\ \mathbf{B}_M K_1 & \mathbf{B}_M K_2 & \mathbf{A}_M + \mathbf{B}_M \mathbf{K}_M & \mathbf{B}_M \mathbf{K}_T \\ [0] & [0] & [0] & \mathbf{A}_T \end{bmatrix} \tag{31}$$

$$\mathbf{A}_{21_{PE}}(t_{go}, \Delta t) = \begin{bmatrix} 0 & 0 & [0] & [0] \\ d_M K_1 & d_M K_2 & \mathbf{C}_M + d_M \mathbf{K}_M & d_M \mathbf{K}_T \\ 0 & 0 & [0] & [0] \end{bmatrix} \tag{32}$$

$$\mathbf{B}_{T_{PE}} = \begin{bmatrix} \mathbf{B}_{MT} + \mathbf{C}_{MT} K_{u_T} \\ \mathbf{C}_{MD} K_{u_T} \end{bmatrix} \quad \mathbf{B}_{D_{PE}} = \begin{bmatrix} [0]_{(2+n_M+n_T)\times 1} \\ \mathbf{B}_{MD} \end{bmatrix} \tag{33}$$

with matrixes $\mathbf{A}_{MD}$ given in Eq. (11) and $\mathbf{B}_{MT}$ with $\mathbf{B}_{MD}$ in Eq. (10) and Eq. (11) accordingly.

B. Cooperative Pursuit Problem Statement

The main objective of the target-defender team is to intercept the attacking missile by the defender prior to its arrival to target, i.e. minimize the miss distance of missile-defender ($|y_{MD}(t_{f_{MD}})|$). Yet, a reasonable control effort is sought from the target and the defender. Thus, we pose the optimal cooperative pursuit problem as the minimization of the following cost function

$$J = \frac{1}{2}\alpha y_{MD}^2(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} u_D^2 + \beta u_T^2 \,\mathrm{d}t \tag{34}$$

subject to the EOM of Eq. (29). Where α and β are non-negative weights.

C. Order Reduction

In order to simplify the solution and to reduce the problem's order we use a transformation from Ref. 15 (denoted by some as *terminal projection*) and define a new variable

$$Z_{MD}(t) = \mathbf{D}\mathbf{\Phi}(t_{f_{MD}}, t)\boldsymbol{x}(t) \tag{35}$$

where $\mathbf{D}$ is a constant vector

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & [0]_{1\times n_M} & [0]_{1\times n_T} & 1 & 0 & [0]_{1\times n_D} \end{bmatrix} \tag{36}$$

and $\mathbf{\Phi}(t_{f_{MD}}, t)$ is a transition matrix associated with Eq. (29), satisfying the fundamental properties of a transition matrix as follows

Remark 1. *Given a linear system with dynamics matrix $\mathbf{A}(t)$, the fundamental properties of associated transition matrix $\mathbf{\Phi}(t_f, t)$ are*

$$\begin{aligned} \dot{\mathbf{\Phi}}(t_f, t) &= -\mathbf{\Phi}(t_f, t)\mathbf{A}(t) \\ \mathbf{\Phi}(t_f, t_f) &= \mathbf{I} \end{aligned} \tag{37}$$

The general form of $\mathbf{\Phi}(t_{f_{MD}}, t)$ for our problem is

$$\mathbf{\Phi}(t_{f_{MD}}, t) = \mathbf{\Phi}(t_{go}) = \begin{bmatrix} \phi_{11} & \phi_{12} & \boldsymbol{\phi}_{1M} & \boldsymbol{\phi}_{1T} & \phi_{15} & \phi_{16} & \boldsymbol{\phi}_{1D} \\ \phi_{21} & \phi_{22} & \boldsymbol{\phi}_{2M} & \boldsymbol{\phi}_{2T} & \phi_{25} & \phi_{26} & \boldsymbol{\phi}_{2D} \\ \boldsymbol{\phi}_{31} & \boldsymbol{\phi}_{32} & \boldsymbol{\phi}_{3M} & \boldsymbol{\phi}_{3T} & \boldsymbol{\phi}_{35} & \boldsymbol{\phi}_{36} & \boldsymbol{\phi}_{3D} \\ \boldsymbol{\phi}_{41} & \boldsymbol{\phi}_{42} & \boldsymbol{\phi}_{4M} & \boldsymbol{\phi}_{4T} & \boldsymbol{\phi}_{45} & \boldsymbol{\phi}_{46} & \boldsymbol{\phi}_{4D} \\ \phi_{51} & \phi_{52} & \boldsymbol{\phi}_{5M} & \boldsymbol{\phi}_{5T} & \phi_{55} & \phi_{56} & \boldsymbol{\phi}_{5D} \\ \phi_{61} & \phi_{62} & \boldsymbol{\phi}_{6M} & \boldsymbol{\phi}_{6T} & \phi_{65} & \phi_{66} & \boldsymbol{\phi}_{6D} \\ \boldsymbol{\phi}_{71} & \boldsymbol{\phi}_{72} & \boldsymbol{\phi}_{7M} & \boldsymbol{\phi}_{7T} & \boldsymbol{\phi}_{75} & \boldsymbol{\phi}_{76} & \boldsymbol{\phi}_{7D} \end{bmatrix} \tag{38}$$

Using the definition of $\mathbf{D}$ and $\mathbf{\Phi}(t_{f_{MD}}, t)$ from Eq. (36) and Eq. (38) we can rewrite Z_{MD} as follows

$$Z_{MD}(t) = [\phi_{51} \; \phi_{52} \; \boldsymbol{\phi}_{5M} \; \boldsymbol{\phi}_{5T} \; \phi_{55} \; \phi_{56} \; \boldsymbol{\phi}_{5D}] \, \boldsymbol{x}(t) \tag{39}$$

The physical meaning of the new state variable Z_{MD}, or zero-effort-miss (ZEM), is being the miss-distance that a defender would achieve if from the current time and onwards not the defender nor the target will apply any control and the missile will continue on it's course using assumed linear pursuit strategy.

To find out the dynamics of Z_{MD} we differentiate it with respect to time and use transition matrix properties from Eq. (37). We obtain Z_{MD} EOM as

$$\dot{Z}_{MD} = \tilde{B}_T(t_{f_{MD}}, t)u_T + \tilde{B}_D(t_{f_{MD}}, t)u_D \tag{40}$$

where

$$\begin{aligned} \tilde{B}_T(t_{f_{MD}}, t) &= \mathbf{D\Phi}(t_{f_{MD}}, t)\mathbf{B}_{T_{PE}} = \phi_{52}\left(d_T - d_M K_{u_T}\right) + \boldsymbol{\phi}_{5M}\mathbf{B}_M K_{u_T} + \boldsymbol{\phi}_{5T}\mathbf{B}_T + \phi_{56} d_M K_{u_T} \\ \tilde{B}_D(t_{f_{MD}}, t) &= \mathbf{D\Phi}(t_{f_{MD}}, t)\mathbf{B}_{D_{PE}} = -\phi_{56} d_D + \boldsymbol{\phi}_{5D}\mathbf{B}_D \end{aligned} \tag{41}$$

We notice that the acquired ZEM dynamics of Eq. (40) is dependent only on defender's and target's controllers u_D and u_T respectively.

We can now reformulate the optimization problem in terms of the new ZEM variable. Since $Z_{MD}(t_{f_{MD}}) = y_{MD}(t_{f_{MD}})$ the cost function of Eq. (34) can be rewritten as

$$J = \frac{1}{2}\alpha Z_{MD}^2(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} u_D^2 + \beta u_T^2 \, \mathrm{d}t \tag{42}$$

and the equivalent reduced-order problem is to minimize the cost function of Eq. (42) subject to the scalar EOM of Eq. (40).

D. Solution

The Hamiltonian of the reduced-order problem is

$$H = \frac{1}{2}\left(u_D^2 + \beta u_T^2\right) + \lambda_Z\left(\tilde{B}_T u_T + \tilde{B}_D u_D\right) \tag{43}$$

The adjoint equation and transversality condition are

$$\begin{cases} \dot{\lambda}_{Z_{MD}} = -\frac{\partial H}{\partial Z_{MD}} = 0 \\ \lambda_{Z_{MD}}(t_{f_{MD}}) = \alpha Z_{MD}(t_{f_{MD}}) \end{cases} \tag{44}$$

The solution of the adjoint equation of Eq. (44) is immediate and it is

$$\lambda_{Z_{MD}}(t) = \alpha Z_{MD}(t_{f_{MD}}) \tag{45}$$

The optimal cooperative pursuit strategies of the defender and the target minimize the Hamiltonian given in Eq. (43) and thus satisfy

$$\begin{aligned} u_T^* &= \arg_{u_T} \min H \\ u_D^* &= \arg_{u_D} \min H \end{aligned} \tag{46}$$

Now, by differentiating the Hamiltonian with respect to each respective controller and equating the result to zero we obtain the open loop optimal controllers for target and defender

$$\begin{aligned} \tfrac{\partial H}{\partial u_T} &= \beta u_T + \lambda_{Z_{MD}} \tilde{B}_T = 0 \\ \tfrac{\partial H}{\partial u_D} &= u_D + \lambda_{Z_{MD}} \tilde{B}_D = 0 \end{aligned} \tag{47}$$

and using the Lagrange multiplier $\lambda_{Z_{MD}}(t)$ from Eq. (45) we finaly obtain

$$\begin{aligned} u_T^* &= -\tfrac{\alpha}{\beta} Z_{MD}(t_{f_{MD}}) \tilde{B}_T \\ u_D^* &= -\alpha Z_{MD}(t_{f_{MD}}) \tilde{B}_D \end{aligned} \tag{48}$$

Then, in order to compute $Z_{MD}(t_{f_{MD}})$ we substitute optimal open loop controllers from Eq. (48) into zero-effort-miss EOM from Eq. (40) and integrate it from current time t to $t_{f_{MD}}$

$$\dot{Z}_{MD}(t) = -\alpha \left(\frac{1}{\beta} \tilde{B}_T^2 + \tilde{B}_D^2 \right) Z_{MD}(t_{f_{MD}}) \tag{49}$$

$$\begin{aligned} Z_{MD}(t_{f_{MD}}) &= Z_{MD}(t) + \textstyle\int_t^{t_{f_{MD}}} \dot{Z}_{MD}(\xi)\, \mathrm{d}\xi = \\ &= Z_{MD}(t) - \alpha Z_{MD}(t_{f_{MD}}) \textstyle\int_t^{t_{f_{MD}}} \tfrac{1}{\beta} \tilde{B}_T^2 + \tilde{B}_D^2 \, \mathrm{d}\xi \end{aligned} \tag{50}$$

We finally obtain $Z_{MD}(t_{f_{MD}})$ as

$$Z_{MD}(t_{f_{MD}}) = \Phi_{Z_{MD}}(t_{f_{MD}}, t) Z_{MD}(t) \tag{51}$$

where

$$\Phi_{Z_{MD}}(t_{f_{MD}}, t) = \frac{1}{1 + \alpha \int_t^{t_{f_{MD}}} \frac{1}{\beta} \tilde{B}_T^2(t_{f_{MD}}, \xi) + \tilde{B}_D^2(t_{f_{MD}}, \xi)\, \mathrm{d}\xi} \tag{52}$$

Substituting $Z_{MD}(t_{f_{MD}})$ from Eq. (51) into open loop controllers from Eq. (48) we obtain optimal two-way cooperative pursuit strategies for defender and target

$$\begin{aligned} u_T^*(t) &= -N_T'(t) Z_{MD}(t)/t_{go}^2 \\ u_D^*(t) &= -N_D'(t) Z_{MD}(t)/t_{go}^2 \end{aligned} \tag{53}$$

where $N_T'(t)$ and $N_D'(t)$ are navigation gains

$$\begin{aligned} N_T'(t) &= \tfrac{\alpha}{\beta} \tilde{B}_T(t_{f_{MD}}, t) \Phi_{Z_{MD}}(t_{f_{MD}}, t) \cdot t_{go}^2 \\ N_D'(t) &= \alpha \tilde{B}_D(t_{f_{MD}}, t) \Phi_{Z_{MD}}(t_{f_{MD}}, t) \cdot t_{go}^2 \end{aligned} \tag{54}$$

In order to complete the solution and implement the derived optimal cooperative strategies for the defender and the target from Eq. (53) we need to compute the transition matrix $\mathbf{\Phi}(t_{f_{MD}}, t)$. Fortunately, the *terminal projection* method we used allows us to reduce these computations only to some necessary elements of the transition matrix, which in our case are ϕ_{51}, ϕ_{52}, $\boldsymbol{\phi}_{5M}$, $\boldsymbol{\phi}_{5T}$, ϕ_{55}, ϕ_{56} and $\boldsymbol{\phi}_{5D}$. Now, using $\mathbf{D}$ from Eq. (36) and the relation from Eq. (37) we can find the differential equation with respect to time-to-go (t_{go}) of these elements

$$\mathbf{D}\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}t_{go}} = -\mathbf{D}\frac{\mathrm{d}\mathbf{\Phi}}{\mathrm{d}t} = \mathbf{D}\mathbf{\Phi}(t_{go})\mathbf{A}_{PE}(t_{go} + \Delta t) \tag{55}$$

with initial condition from Eq. (37) as

$$\mathbf{\Phi}(t_{go} = 0) = \mathbf{I} \tag{56}$$

Substituting $\mathbf{D}$ and $\mathbf{A}_{PE}(t_{go} + \Delta t)$ we obtain

$$\left\{ \begin{aligned} \frac{d\phi_{51}}{dt_{go}} &= -d_M K_1 \phi_{52} + K_1 \boldsymbol{\phi}_{5M} \mathbf{B}_M + d_M K_1 \phi_{56} \\ \frac{d\phi_{52}}{dt_{go}} &= \phi_{51} - d_M K_2 \phi_{52} + K_2 \boldsymbol{\phi}_{5M} \mathbf{B}_M + d_M K_2 \phi_{56} \\ \frac{d\boldsymbol{\phi}_{5M}}{dt_{go}} &= [-\mathbf{C}_M - d_M \mathbf{K}_M]\phi_{52} + \boldsymbol{\phi}_{5M}[\mathbf{A}_M + \mathbf{B}_M \mathbf{K}_M] + \phi_{56}(\mathbf{C}_M + d_M \mathbf{K}_M) \\ \frac{d\boldsymbol{\phi}_{5T}}{dt_{go}} &= [\mathbf{C}_T - d_M \mathbf{K}_T]\phi_{52} + \boldsymbol{\phi}_{5M} \mathbf{B}_M \mathbf{K}_T + \boldsymbol{\phi}_{5T} \mathbf{A}_T + d_M \mathbf{K}_T \phi_{56} \\ \frac{d\phi_{55}}{dt_{go}} &= 0 \\ \frac{d\phi_{56}}{dt_{go}} &= \phi_{55} \\ \frac{d\boldsymbol{\phi}_{5D}}{dt_{go}} &= -\phi_{56} \mathbf{C}_D + \boldsymbol{\phi}_{5D} \mathbf{A}_D \end{aligned} \right. \qquad \begin{aligned} \phi_{51}(0) &= 0 \\ \phi_{52}(0) &= 0 \\ \boldsymbol{\phi}_{5M}(0) &= [0] \\ \boldsymbol{\phi}_{5T}(0) &= [0] \\ \phi_{55}(0) &= 1 \\ \phi_{56}(0) &= 0 \\ \boldsymbol{\phi}_{5D}(0) &= [0] \end{aligned} \tag{57}$$

Once the endgame phase of the engagement is entered Eq. (57) can be solved numerically as a function of time-to-go and cooperative guidance laws can be implemented. The solution of ϕ_{55} and ϕ_{56} is immediate and it is

$$\begin{aligned} \phi_{55}(t_{go}) &= 1 \\ \phi_{55}(t_{go}) &= t_{go} \end{aligned} \tag{58}$$

And if the missile is implementing a PN, APN or OGL guidance, where the relation $K_2 = (t_{go} + \Delta t) \cdot K_1$ holds, it can be shown that

$$\phi_{52} = \phi_{51} \cdot (t_{go} + \Delta t) \tag{59}$$

E. Special Cases

The obtained cooperative strategies from Eq. (53) are functions of the weights α and β. These weights represent the relative penalty on a miss distance (α) and the relative penalty on the target's control effort (β). Note that in the cost function of Eq. (42) the penalty on the defender's control effort is 1. Thus, in order to achieve perfect interception of the missile by the defender we have to vastly increase the weight on the miss distance α compared to the weights on the control efforts β and 1 i.e. we require $\alpha \to \infty$. Therefore, the optimal controllers remain as in Eq. (53) and the respective navigation gains are obtained by inducing $\alpha \to \infty$ on Eq. (54)

$$\begin{aligned} N'_{T,\alpha\to\infty}(t) &= \frac{\tilde{B}_T(t_{f_{MD}},t)}{\beta \int_t^{t_{f_{MD}}} \frac{1}{\beta}\tilde{B}_T^2(t_{f_{MD}},\xi)+\tilde{B}_D^2(t_{f_{MD}},\xi)\,\mathrm{d}\xi} \cdot t_{go}^2 \\ N'_{D,\alpha\to\infty}(t) &= \frac{\tilde{B}_D(t_{f_{MD}},t)}{\int_t^{t_{f_{MD}}} \frac{1}{\beta}\tilde{B}_T^2(t_{f_{MD}},\xi)+\tilde{B}_D^2(t_{f_{MD}},\xi)\,\mathrm{d}\xi} \cdot t_{go}^2 \end{aligned} \tag{60}$$

In case where in addition to perfect interception the target has limited or no maneuvering capability, e.g. passenger aircraft, the weight on target's control effort must be increased along with the weight on the miss distance. Thus, the corresponding navigation gains are obtained by inducing infinite weight on target's control effort, i.e. $\beta \to \infty$, together with infinite weight on the miss distance, i.e. $\alpha \to \infty$,

$$\begin{aligned} N'_{T,\alpha\to\infty,\beta\to\infty}(t) &= 0 \\ N'_{D,\alpha\to\infty,\beta\to\infty}(t) &= \frac{\tilde{B}_D(t_{f_{MD}},t)}{\int_t^{t_{f_{MD}}} \tilde{B}_D^2(t_{f_{MD}},\xi)\,\mathrm{d}\xi} \cdot t_{go}^2 \end{aligned} \tag{61}$$

For the case where perfect interception is required from a non-maneuvering defender we induce an infinite weight on defender's control effort in addition to the weight on the miss distance, i.e. $\beta \to 0$ and $\alpha \to \infty$. And we obtain the navigation gains as follows,

$$\begin{aligned} N'_{T,\alpha\to\infty,\beta\to 0}(t) &= \frac{\tilde{B}_T(t_{f_{MD}},t)}{\int_t^{t_{f_{MD}}} \tilde{B}_T^2(t_{f_{MD}},\xi)\,\mathrm{d}\xi} \cdot t_{go}^2 \\ N'_{D,\alpha\to\infty,\beta\to 0}(t) &= 0 \end{aligned} \tag{62}$$

IV. Cooperative Target Strategy with Independent Defender

In the previous section we derived optimal strategies for a general case where two-way information sharing allowed the target and the defender to act cooperatively as a team against a homing missile. However, such a two-way communication requires additional hardware installed on the defender, and may be often unavailable at hand. In this section we consider a case where a different cooperation scheme is employed, assuming that only one-way information sharing is possible. For example, the defender can be driven independently using an existing one-on-one strategy while transmitting the data on its future maneuver to the target. Using the information on defender's strategy, along with the knowledge on the strategy of the missile the target can predict the behavior of the defender and bring the missile to interception with the defender. As a realistic example of such a scenario we can think of an aircraft equipped with already existing missiles where a guidance law such as PN is implemented. Thus, in the current section we derive a one-way cooperative support strategy for the target to aid the defender hit the missile. We assume that the defender missile is driven by a one-on-one linear guidance law of the form described in Eq. (18) (such as PN). The underlying assumption is that the missile's and defender's guidance strategies are known to the target. It is logical to assume a known defender's strategy as it is fired from the target, while missile's guidance law may be identified using a technique such as that proposed in the first year of the research.[11]

A. Engagement Dynamics

We assume that both missile and defender employ linear guidance strategies. The corresponding notation for the missile's strategy was shown in Eq. (18). Thus, we can write the defender's guidance law as follows

$$u_D = \mathbf{K}_D(t_{go_{MD}})[\boldsymbol{x}_{MD}^T\ \boldsymbol{x}_M^T]^T + K_{u_{T_D}}(t_{go_{MD}})u_M \tag{63}$$

where

$$\mathbf{K}_D\left(t_{go_{MD}}\right) = \left[\begin{array}{cccc} K_{1_D} & K_{2_D} & \mathbf{K}_{M_D} & \mathbf{K}_{T_D} \end{array}\right] \tag{64}$$

By substituting missile's and defender's guidance laws given in Eq. (18) and Eq. (63) into Eq. (8) we obtain the EOM of the cooperative one-way single controller evasion problem:

$$\dot{\boldsymbol{x}} = \mathbf{A}_E(t_{go}, \Delta t)\boldsymbol{x} + \mathbf{B}_E u_T \tag{65}$$

where

$$\mathbf{A}_E(t_{go}, \Delta t) = \begin{bmatrix} \mathbf{A}_{MT_{PE}}(t_{go} + \Delta t) & [0] \\ \mathbf{A}_{21_E}(t_{go}, \Delta t) & \mathbf{A}_{MD_E}(t_{go}) \end{bmatrix}, \quad \mathbf{B}_E = \begin{bmatrix} \mathbf{B}_{MT_E}(t_{go}) \\ \mathbf{B}_{MD_E}(t_{go}, \Delta t) \end{bmatrix} \tag{66}$$

$$\mathbf{A}_{21_E}(t_{go}, \Delta t) = \begin{bmatrix} 0 & 0 & [0] & [0] \\ GK_1 & GK_2 & \mathbf{C}_M + G\mathbf{K}_M - d_D\mathbf{K}_{T_D} & G\mathbf{K}_T \\ \mathbf{H}K_1 & \mathbf{H}K_2 & (\mathbf{H}\mathbf{K}_M + \mathbf{B}_D\mathbf{K}_{T_D}) & \mathbf{H}\mathbf{K}_T \end{bmatrix} \tag{67}$$

where

$$G = (d_M - K_{u_{T_D}} d_D); \quad \mathbf{H} = K_{u_{T_D}} \mathbf{B}_D \tag{68}$$

and

$$\mathbf{A}_{MD_E}(t_{go}) = \begin{bmatrix} 0 & 1 & 0 \\ -d_D K_{1_D} & -d_D K_{2_D} & -(\mathbf{C}_D + d_D \mathbf{K}_{M_D}) \\ \mathbf{B}_D K_{1_D} & \mathbf{B}_D K_{2_D} & \mathbf{A}_D + \mathbf{B}_D \mathbf{K}_{M_D} \end{bmatrix} \tag{69}$$

$$\mathbf{B}_{MT_E} = \begin{bmatrix} 0 \\ d_T - K_{u_T} d_M \\ \mathbf{B}_M K_{u_T} \\ \mathbf{B}_T \end{bmatrix} \quad \mathbf{B}_{MD_E} = \begin{bmatrix} 0 \\ -d_D K_{u_T} K_{u_{T_D}} + d_M K_{u_T} \\ \mathbf{B}_D K_{u_T} K_{u_{T_D}} \end{bmatrix} \tag{70}$$

B. Problem Statement

To achieve an interception of the missile by the defender the primary objective of the target during the missile-defender engagement is to minimize the miss distance between missile and defender ($|y_{MD}(t_{f_{MD}})|$) and yet apply a reasonable maneuver. The cost function that describes the above objectives is therefore

$$J_1 = \frac{1}{2}\alpha y_{MD}^2(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} u_T^2 \, \mathrm{d}t \tag{71}$$

subject to the EOM of Eq. (65).

However, in the above cost function formulation there is no penalty set for the control effort of the defender whereas its linear guidance law was derived assuming unbounded control. As a consequence, the solution of the problem will provide us with a target strategy that answers the objectives defined by the cost function, but at the same time it may result in excessive defender's acceleration. In a realistic scenario where defender's acceleration is limited by a finite value it will saturate causing a miss distance. Consequently, the target must take into account also the minimization of the defender's control effort. Thus, the respective cost function will be as follows

$$J_2 = \frac{1}{2}\alpha y_{MD}^2(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} u_D^2 + \beta u_T^2 \,\mathrm{d}t \tag{72}$$

where α is a non-negative weight representing a penalty on the miss distance and β is a non-negative weight representing a relative penalty between the control efforts of the defender and the target.

Apparently, the second formulation of Eq. (72) represents a more general case of the first one of Eq. (71). Thus, we continue to focus on a solution of the second formulation. In addition, since all linear guidance laws that may be implemented by the defender are designed for interception (i.e. minimize the missile-defender miss distance), once the target begins to actively help the defender (i.e. by reducing defenders control effort) a hit-to-kill performance will be guarantied. Thus, the weight on the miss distance for the second formulation may not be necessary (i.e. $\alpha = 0$), as will be shown in the simulation section.

C. Solution

Using the defender's controller from Eq. (63) and substituting a missile controller from Eq. (18) we can reformulate the cost function given in Eq. (72) to the following linear quadratic form:

$$J_2 = \frac{1}{2}\boldsymbol{x}^T(t_{f_{MD}})\mathbf{Q}_f\boldsymbol{x}(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} \boldsymbol{x}^T\mathbf{Q}(t_{go})\boldsymbol{x} + 2u_T\mathbf{L}\boldsymbol{x} + u_T^T R u_T \,\mathrm{d}t \tag{73}$$

where

$$\mathbf{Q}_f = \begin{bmatrix} [0]_{(2+n_M+n_T)\times(2+n_M+n_T)} & [0]_{(2+n_M+n_T)\times 1} & [0]_{(2+n_M+n_T)\times(1+n_D)} \\ [0]_{1\times(2+n_M+n_T)} & \alpha & [0]_{1\times(1+n_D)} \\ [0]_{(1+n_D)\times(2+n_M+n_T)} & [0]_{(1+n_D)\times 1} & [0]_{(1+n_D)\times(1+n_D)} \end{bmatrix} \tag{74}$$

$$\mathbf{Q} = \mathbf{V}^T\mathbf{V}; \quad \mathbf{L} = K_{u_{T_D}}K_{u_T}\mathbf{V}; \quad R = (K_{u_{T_D}}K_{u_T})^2 + \beta \tag{75}$$

and

$$\mathbf{V} = \begin{bmatrix} K_{u_{T_D}}K_1 & K_{u_{T_D}}K_2 & K_{u_{T_D}}\mathbf{K}_M + \mathbf{K}_{T_D} & K_{u_{T_D}}\mathbf{K}_T & K_{1_D} & K_{2_D} & \mathbf{K}_{M_D} \end{bmatrix} \tag{76}$$

The Hamiltonian of the problem is

$$H = \frac{1}{2}\mathbf{x}^T\mathbf{Q}(t_{go})\mathbf{x} + u_T\mathbf{L}\mathbf{x} + \frac{1}{2}u_T^T R u_T + \boldsymbol{\lambda}^T\mathbf{A}_E\mathbf{x} + \boldsymbol{\lambda}^T\mathbf{B}_E u_T \tag{77}$$

The adjoint equation and transversality condition are

$$\begin{cases} \dot{\boldsymbol{\lambda}} = -\left(\frac{\partial H}{\partial \boldsymbol{x}}\right)^T = -\mathbf{Q}(t_{go})\boldsymbol{x} - \mathbf{L}^T u_T - {\mathbf{A}_E}^T\boldsymbol{\lambda}(t) \\ \boldsymbol{\lambda}(t_{f_{MD}}) = \mathbf{Q}_f\boldsymbol{x}(t_{f_{MD}}) \end{cases} \tag{78}$$

The optimal cooperative target strategy minimizes the Hamiltonian and thus satisfies

$$u_T^* = \arg_{u_T}\min H = \arg_{u_T}\{\tfrac{\partial H}{\partial u_T} = \mathbf{L}\boldsymbol{x}Ru_T + \mathbf{B}_E^T\boldsymbol{\lambda} = 0\} \tag{79}$$

$$u_T^* = -R^{-1}\left(\mathbf{B}_E^T\boldsymbol{\lambda}(t) + \mathbf{L}\mathbf{x}^*\right) \tag{80}$$

where $\boldsymbol{x}^*$ is the optimal trajectory.

The solution of the adjoint equation given in Eq. (78) with implemented optimal controller from Eq. (79) is given by

$$\boldsymbol{\lambda}(t) = \boldsymbol{P}(t)\boldsymbol{x}(t) \tag{81}$$

where $\boldsymbol{P}(t)$ is a symmetric matrix of the following form

$$\boldsymbol{P}(t) = \begin{bmatrix} p_{11} & p_{12} & \mathbf{p}_{1M} & \mathbf{p}_{1T} & p_{15} & p_{16} & \mathbf{p}_{1D} \\ p_{12} & p_{22} & \mathbf{p}_{2M} & \mathbf{p}_{2T} & p_{25} & p_{26} & \mathbf{p}_{2D} \\ \mathbf{p}_{1M}^T & \mathbf{p}_{2M}^T & \mathbf{p}_{3M} & \mathbf{p}_{3T} & \mathbf{p}_{35} & \mathbf{p}_{36} & \mathbf{p}_{3D} \\ \mathbf{p}_{1T}^T & \mathbf{p}_{2T}^T & \mathbf{p}_{3T}^T & \mathbf{p}_{4T} & \mathbf{p}_{45} & \mathbf{p}_{46} & \mathbf{p}_{4D} \\ p_{15} & p_{25} & \mathbf{p}_{35}^T & \mathbf{p}_{45}^T & p_{55} & p_{56} & \mathbf{p}_{5D} \\ p_{16} & p_{26} & \mathbf{p}_{36}^T & \mathbf{p}_{46}^T & p_{56} & p_{66} & \mathbf{p}_{6D} \\ \mathbf{p}_{1D}^T & \mathbf{p}_{2D}^T & \mathbf{p}_{3D}^T & \mathbf{p}_{4D}^T & \mathbf{p}_{5D}^T & \mathbf{p}_{6D}^T & \mathbf{p}_{7D} \end{bmatrix} \tag{82}$$

satisfying the following differential matrix Riccati equation

$$\begin{cases} \dot{\mathbf{P}} = -\left(\mathbf{P}\mathbf{A}_E + \mathbf{A}_E^T\mathbf{P} - \mathbf{P}\mathbf{B}_E R^{-1}\mathbf{B}_E^T\mathbf{P} - 2\mathbf{P}\mathbf{B}_E R^{-1} - \mathbf{L}^T R^{-1}\mathbf{L} + \mathbf{Q}\right) \\ \mathbf{P}(t_{f_{MD}}) = \mathbf{Q}_f \end{cases} \tag{83}$$

And the closed loop optimal evasion strategy is therefore

$$u_T^*(t) = -R^{-1}\left(\mathbf{B}_E^T\boldsymbol{P}(t) + \mathbf{L}(t)\right)\boldsymbol{x}(t) \tag{84}$$

We can now substitute R , $\mathbf{B}_E$ and $\mathbf{P}(t)$ from Eq. (75), Eq. (70) and Eq. (82) respectively and obtain

$$u_T^*(t) = -\frac{\mathbf{K}_E^*(t)}{(K_{u_{T_D}} K_{u_T})^2 + \beta} \boldsymbol{x}(t) \tag{85}$$

where

$$\mathbf{K}_E^*(t) = \begin{bmatrix} d_T p_{12} + \mathbf{B}_T^T \mathbf{p}_{1T}^T + K_{u_{T_D}}^2 K_{u_T} K_1 \\ d_T p_{22} + \mathbf{B}_T^T \mathbf{p}_{2T}^T + K_{u_{T_D}}^2 K_{u_T} K_2 \\ d_T \mathbf{p}_{2M} + \mathbf{B}_T^T \mathbf{p}_{3T}^T + K_{u_{T_D}}^2 K_{u_T} \mathbf{K}_M + K_{u_{T_D}} K_{u_T} \mathbf{K}_{T_D} \\ d_T \mathbf{p}_{2T} + \mathbf{B}_T^T \mathbf{p}_{4T} + K_{u_{T_D}}^2 K_{u_T} \mathbf{K}_T \\ d_T p_{25} + \mathbf{B}_T^T \mathbf{p}_{45} + K_{u_{T_D}} K_{u_T} K_{1_D} \\ d_T p_{26} + \mathbf{B}_T^T \mathbf{p}_{46} + K_{u_{T_D}} K_{u_T} K_{2_D} \\ d_T \mathbf{p}_{2D} + \mathbf{B}_T^T \mathbf{p}_{4D} + K_{u_{T_D}} K_{u_T} \mathbf{K}_{M_D} \end{bmatrix}^T \tag{86}$$

In order to implement this strategy the matrix $\mathbf{P}$ shall be calculated prior to the endgame phase of the engagement. Since we can not obtain a closed form solution for $\mathbf{P}$ it will be calculated numerically.

V. Cooperative Defender Strategy with Independent Target

In the previous section we analyzed a target-defender team actions under the constraint of independently guided defender and presented a corresponding one-way cooperation scheme. Now, we turn our attention to the second one-way cooperation scheme, where the information is transferred from the target to the defender. We assume that the target may perform any arbitrary maneuver and transmits the data regarding its chosen strategy to the defender. For example, an aircraft pilot may fire a defender on the incoming missile and perform an independent evasive maneuver. Based on the oncoming maneuvering sequence received from the target, together with the knowledge of missile's guidance law, the defender is able to predict the missile's behavior and head towards the interception point. Thus, an appropriate one-way cooperative pursuit strategy is required for the defender. As in the previous section we apply an assumption that the attacking missile uses a known linear guidance law of the form given in Eq. (18). We also assume that the target performs a known arbitrary maneuvering strategy. For example, since the missile's guidance law is known it is natural for the target to employ an optimal one-on-one evasive maneuver. The corresponding one-on-one evasion strategies assuming bounded target control and a pursuer implementing a linear guidance were derived in the second year of the research[12] and have the following form

$$u_{T_E}^* = u_T^{max} \operatorname{sign}(s_{MT}) \operatorname{sign}(Z_{MT}) \tag{87}$$

where s_{MT} is the appropriate switching function and Z_{MT} is the first component of the homogeneous solution of missile-target engagement.

A. Pursuit Dynamics

The missile-target-defender engagement equations of motion remain as given in Eq. (8). Following the assumption that the missile uses a linear guidance strategy we substitute Eq. (18) into Eq. (8) to obtain the EOM of pursuit-evasion problem as it was presented in Eq. (30). We also assume that the target performs a known evasive maneuver, such as that given in Eq. (87). Thus, the EOM are

$$\dot{\boldsymbol{x}} = \mathbf{A}_{PE}(t_{go}, \Delta t)\boldsymbol{x} + \mathbf{B}_{T_{PE}} u^*_{T_E} + \mathbf{B}_{D_{PE}} u_D \tag{88}$$

B. Pursuit Problem Statement

The defender's objective during the engagement is to ensure the safety of the target by intercepting the homing missile. This can be achieved by minimizing the miss distance ($|y_{MD}(t_{f_{MD}})|$), while not exceeding defender's maneuvering capabilities. Thus, we pose the cooperative pursuit problem as the minimization of the following cost function

$$J = \frac{\alpha}{2} y^2_{MD}(t_{f_{MD}}) + \frac{1}{2} \int_0^{t_{f_{MD}}} u^2_D \, \mathrm{d}t \tag{89}$$

subject to EOM from Eq. (88) and with α being a non-negative weight which is a relative penalty between the miss distance and the control effort.

C. Order Reduction

In order to simplify the solution and to reduce the problem's order we again use the *terminal projection* method and define a new variable

$$Z_{MD_P}(t) = \mathbf{D}\boldsymbol{\Phi}(t_{f_{MD}}, t)\boldsymbol{x}(t) + \mathbf{D} \int_t^{t_{f_{MD}}} \boldsymbol{\Phi}(t_{f_{MD}}, \xi)\mathbf{B}_{T_{PE}} u^*_{T_E}(\xi) \, \mathrm{d}\xi \tag{90}$$

where $\mathbf{D}$ is a constant vector given in Eq. (36) and $\boldsymbol{\Phi}(t_{f_{MD}}, t)$ is the transition matrix associated with Eq. (88). Since in the current engagement formulation the dynamics matrix is the same as in Eq. (29) the resulting transition matrix is identical to that given in Eq. (38) and satisfies Eq. (37). The general form of $\boldsymbol{\Phi}(t_{f_{MD}}, t)$ for our problem was presented earlier and is given in Eq. (38).

Using the definition of $\mathbf{D}$ and $\boldsymbol{\Phi}(t_{f_{MD}}, t)$ from Eq. (36) and Eq. (38), and $\mathbf{B}_{T_{PE}}$ from

Eq. (33) we can rewrite Z_{MD} as follows

$$Z_{MD_P}(t) = [\phi_{51}\ \phi_{52}\ \boldsymbol{\phi}_{5M}\ \boldsymbol{\phi}_{5T}\ \phi_{55}\ \phi_{56}\ \boldsymbol{\phi}_{5D}]\,\boldsymbol{x}(t) + \int_t^{t_{f_{MD}}} (\phi_{52} d_T + \boldsymbol{\phi}_{5T}\mathbf{B}_T) u^*_{T_E}(\xi)\,\mathrm{d}\xi \tag{91}$$

The physical meaning of the new state variable Z_{MD} or zero-effort-miss (ZEM) is a miss-distance that a defender would achieve if from the current time and onwards it won't apply any control and the target and missile will continue on their course using their assumed pursuit and evasion strategies.

To find out the dynamics of Z_{MD_P} we again differentiate it with respect to time and use Eq. (37). And we obtain Z_{MD_P} EOM as

$$\dot{Z}_{MD_P} = \tilde{B}_D(t_{f_{MD}}, t) u_D \tag{92}$$

where $\tilde{B}_D$ is given in Eq. (33). We notice that the acquired ZEM dynamics of Eq. (92) is dependent only on defender's controller u_D, as the target is employing a pre-decided upon maneuver.

We can now reformulate the optimization problem in terms of the ZEM variable. Since $Z_{MD}(t_{f_{MD}}) = y_{MD}(t_{f_{MD}})$ the cost function of Eq. (89) can be rewritten as

$$J = \frac{\alpha}{2} Z^2_{MD_P}(t_{f_{MD}}) + \frac{1}{2}\int_0^{t_{f_{MD}}} u_D^2\,\mathrm{d}t \tag{93}$$

and the equivalent reduced-order problem is to minimize the cost function of Eq. (93) subject to the EOM of Eq. (92).

D. Solution

The Hamiltonian of the reduced-order problem is

$$H = \frac{1}{2} u_D^2 + \lambda_{Z_{MD_P}} \tilde{B}_D(t_{f_{MD}}, t) u_D \tag{94}$$

The adjoint equation and transversality condition are

$$\begin{cases} \dot{\lambda}_{Z_{MD_P}} = -\frac{\partial H}{\partial Z_{MD_P}} = 0 \\ \lambda_{Z_{MD_P}}(t_{f_{MD}}) = Z_{MD_P}(t_{f_{MD}}) \end{cases} \tag{95}$$

Following the same steps as it was done in section III we obtain the optimal cooperative pursuit strategy for the defender

$$u_D^*(t) \;\; = -N'_{D_P}(t) Z_{MD_P}(t)/t^2_{go} \tag{96}$$

where $N'_{D_P}(t)$ is the navigation gain satisfying

$$N'_{D_P}(t) \quad = \alpha \tilde{B}_D(t_{f_{MD}}, t) \Phi_{Z_{MD_P}}(t_{f_{MD}}, t) \cdot t_{go}^2 \tag{97}$$

and

$$\Phi_{Z_{MD_P}}(t_{f_{MD}}, t) = \frac{1}{1 + \alpha \int_t^{t_{f_{MD}}} \tilde{B}_P^2(t_{f_{MD}}, \xi)\, \mathrm{d}\xi} \tag{98}$$

For perfect interception we choose $\alpha \to \infty$ as it was done in previous sections and obtain the navigation gain

$$N'_{D_{P,\alpha \to \infty}}(t) \quad = \frac{\tilde{B}_D(t_{f_{MD}}, t)}{\int_t^{t_{f_{MD}}} \tilde{B}_P^2(t_{f_{MD}}, \xi)\, \mathrm{d}\xi} \cdot t_{go}^2 \tag{99}$$

VI. Simulation Analysis

In this section we investigate the performance of the derived cooperation schemes via analysis of simulation results. Each of the three proposed cooperation schemes is discussed separately and then compared. Such a comparison enables us to analyze the impact of cooperation between target and defender on the outcome of the engagement. For the simulations we assume that missile, target, and defender have first order acceleration dynamics with identical time constants of 0.1 seconds. The missile employs PN guidance and we assume that this strategy is known to the adversaries. The duration of the missile-defender engagement is 0.7 seconds, and the time difference Δt between the terminal times of missile-target and missile-defender engagements is 0.3 seconds.

A. Two-Way Cooperative Pursuit

In this subsection we examine the first and more general case where the target and the defender act in full cooperation by means of two-way information sharing to intercept the incoming missile. Once the endgame phase of the engagement has begun and the missile's guidance strategy was identified then the target and defender may switch to their respective cooperative strategies. In order to implement the optimal two-way cooperative pursuit guidance laws of Eq. (53) the relevant elements of the transition matrix must be computed by solving the system of differential equations Eq. (57) (i.e. numerical solution). Next step in the implementation is to assign penalty weights α and β according to desired performance or other engagement constraints (i.e. acceleration limitations). In Fig. 2 we present the *Pareto Front* of the problem as a function of weights α and β. Each point on the surface depicted on the figure represents a simulation result in terms of miss distance $y_{MD}(t_f)$ and control effort of the target ($\int u_T^2\, \mathrm{d}t$) and the defender ($\int u_D^2\, \mathrm{d}t$). When the target and the defender both use optimal strategies the point representing the outcome of the engagement lies on the *Pareto Front* surface. It is therefore not possible to gain a better result, i.e. descend bellow the *Pareto Front* by deviating from these optimal strategies. Thus, if for any reason a different strategy is chosen by the target or the defender, the resulting outcome will be above the surface. As expected, it can be observed that increasing the penalty on miss distance, i.e. $\alpha \to \infty$, reduces $y_{MD}(t_f)$. However, at the same time for a given β the control efforts of the target and the defender increase as they are required to perform harder to achieve smaller miss. We also notice that by increasing the relative penalty on control effort β we obtain reduced control effort for target and increased effort for the defender. For a given α, increasing β will also cause the miss distance $y^2_{MD}(t_f)$ to increase, since target tries less to assist the defender.

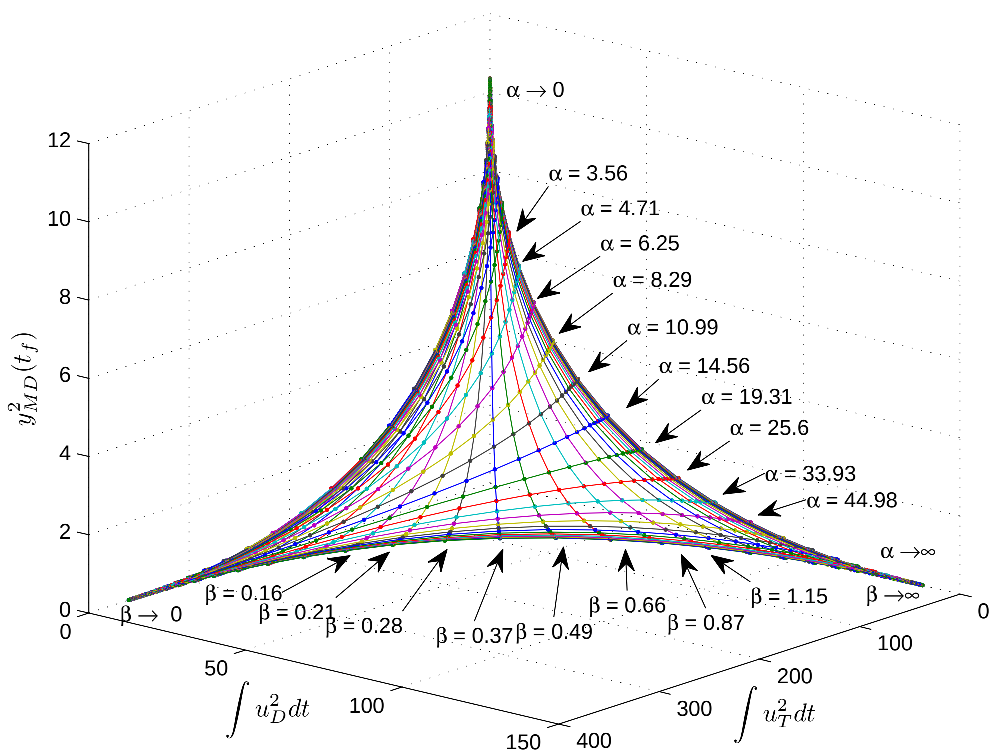

Figure 2. Cooperative Pursuit Pareto Front as a function of α and β.

The part of the *Pareto Front* for the limiting case where we want to assure interception of the missile, i.e. $\alpha \to \infty$, is presented in Fig. 3. By decreasing β we place less penalty on target's maneuvering, thus allowing the target to more actively help the defender . As a consequence the control effort required from the defender decreases. Actually, this limiting case *Pareto Front* is a cross section of the general case *Pareto Front* surface shown in Fig. 2 at $y^2_{MD}(t_f) = 0$.

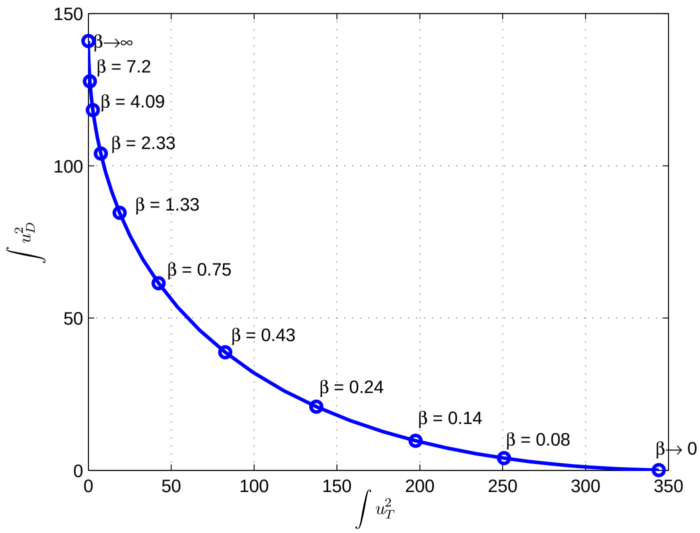

Figure 3. Pareto Front for the limiting perfect interception case where $\alpha \to \infty$ as a function of β.

The time evolution of navigational gains N'_T and N'_D for various weights β is plotted in Fig. 4. Decreasing β raises the navigation gain of the target N'_T, since by decreasing the penalty on the target maneuver, we are actually allowing an increase in the control; the same is true about the defender's navigation gain N'_D. The resulting optimal trajectories over a set of weights β is plotted in Fig. 5. During the engagement cooperatively acting target and defender cause Z_{MD} to monotonically decrease until it's nulled.

Note, an outcome of a scenario where the target and the defender both use existing one-on-one strategies can be presented on Fig. 2 as a single point. For example, in a case where the defender employs PN guidance and the target performs a constant maneuver an interception is achieved. Therefore, the point will be on a plane corresponding to $y_{MD}(t_f) = 0$ and we can refer to Fig. 3. In addition, the control effort of the target equals to some positive value and that of the defender tends to a very large number (i.e. $\int u_T^2 dt = 10^{35}$). Thus, the corresponding point representing such an outcome is beyond the figure boundaries and above the *Pareto Front* line. It is evident that using such a one-on-one strategies yields worse performance than by cooperative actions.

B. Cooperative Target With Independent Defender

Now we turn our attention to the second guidance law derived for the target to cooperatively aid the defender intercept the missile. Recall that in this case we rely on a one-way information sharing set from the defender to the target, i.e. the defender is independently guided by some linear strategy and transmits this data to the target. This information is then used by the target to bring the defender to collision with the missile. The respective

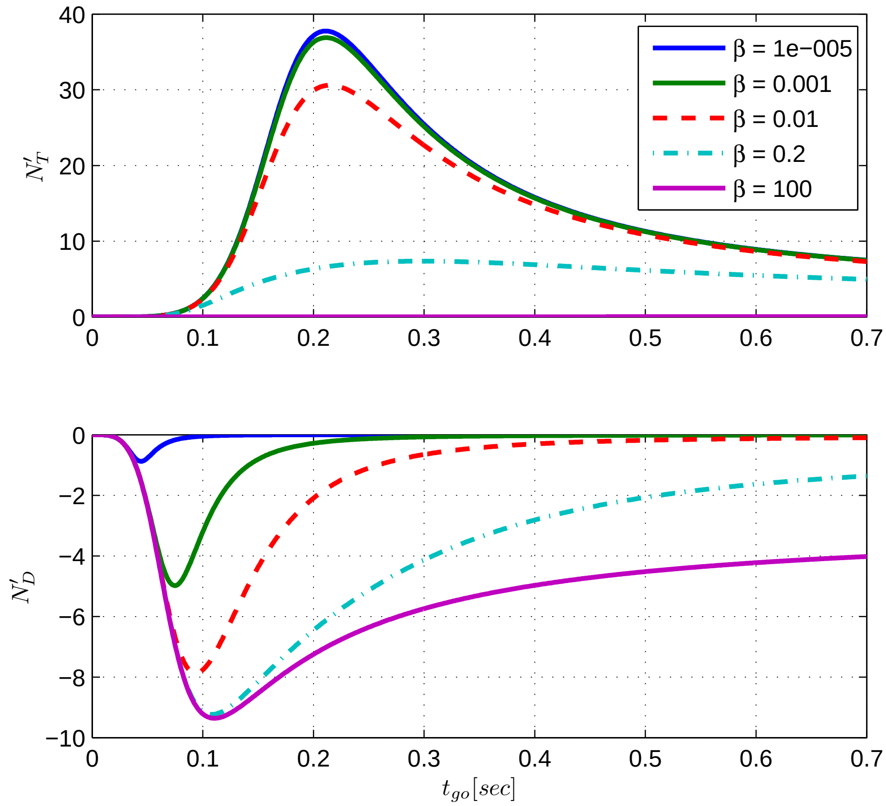

Figure 4. Navigation gains evolution for two-way cooperation.

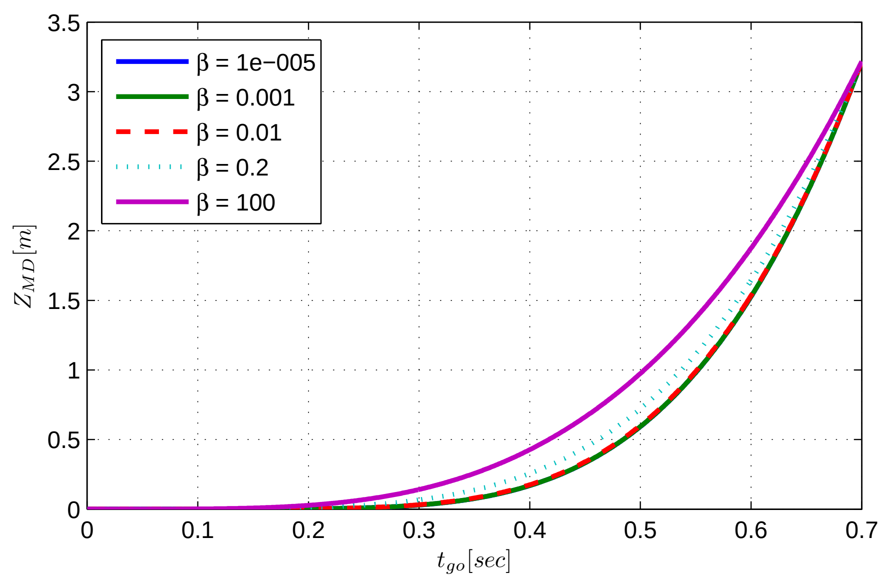

Figure 5. Z_{MD} evolution for perfect interception case.

strategy is given in Eq. (85). We assume that the missile and the defender are guided using PN guidance law with effective gain $N' = 3$ that is known to the target.

To implement the guidance law, the relative penalty weight on the miss distance α and on the target control effort β must be assigned first. Next, the respective elements of matrix $\mathbf{P}$ must be computed according to the solution of Eq. (83).

The simulation was set over the same engagement's initial conditions as in the previous case. The resulting *Pareto Front* is shown in Fig. 6. We present the results only for $\alpha = 0$, as identical results were obtained for other various values of α. Note, that for the current set of initial conditions and due to unbounded control the defender manages to achieve a zero miss distance even without the target's assistance (i.e. $u_T = 0$). Since the penalty on miss distance does not affect the outcome and a perfect interception is achieved it is clear that the target maneuvers only to reduce the control effort of the defender. In Fig. 6 we also depict the cross section of the cooperative pursuit *Pareto Front* at $y_{MD}^2(t_f) = 0$ from Fig. 2, which is also shown in Fig. 3. The control efforts required from the target and the defender when using full cooperation are considerably smaller than in the present case; therefore, this cross section appears as a point at the origin. Apparently, the obtained *Pareto Front* of the current cooperation scheme is significantly above the *Pareto Front* of the two-way cooperation case, since the strategies used by the target and the defender in the current scenario considerably deviate from the optimal two-way cooperative pursuit strategies. Despite the fact that the target's strategy is optimal for the current scenario it does not exploit the full potential of cooperation due to the constraint on the guidance law of the defender.

When the penalty on target's control effort β is very large (greater than 10^{20}), the target does not perform any maneuver and it's control effort equals zero. In this case, the defender is acting alone in absence of aid from the target. As a consequence, the required defender control effort is huge. Such an outcome is a result of PN guidance implemented on a defender with non ideal dynamics. Once the penalty decreases, and the target is allowed to maneuver it manages to significantly decrease the control effort required from the defender. However, to improve the outcome and to further reduce the control effort of the defender the target must considerably increase its own control effort. It is evident that there is a diminishing return for target maneuvers. .

It is important to note that the main contribution of the one-way cooperation here is the possibility to reduce the required control effort of the defender, since the interception of the missile could have been achieved even without the target's assistance. However, it is clearly seen by comparing the results shown in Fig. 3 and Fig. 6, that a one-way cooperation scheme requires a considerably more agile target. In order to be able to support the defender with the preset one-on-one guidance law, the target and the defender must have much superior maneuvering capabilities compared to the two-way cooperation case.

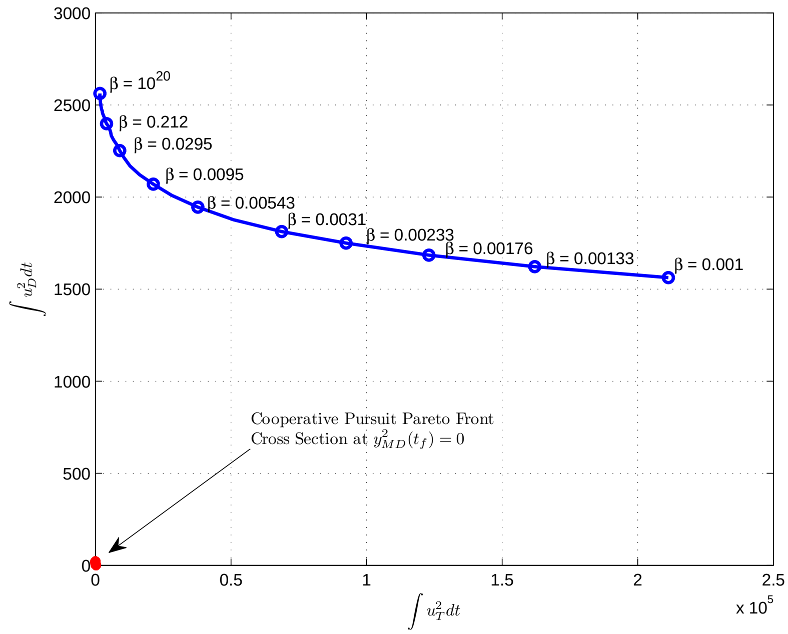

Figure 6. Pareto Front for cooperative target with independent defender.

The guidance law derived for the current cooperation scheme, as shown in Eq. (85) has a different structure than the general two-way cooperation case guidance strategy, as shown in Eq. (53).The current controller is a function of time dependent gains vector multiplied by the current state vector, whereas two-way cooperation controllers have a form of ZEM multiplied by a navigational gain. Thus, there is no clear way to compare those gains and therefore they are not shown here.

C. Cooperative Defender with Independent Target

In this subsection we analyze the performance of cooperatively acting defender with independently evading target. The guidance law is given in Eq. (96). The underlying assumption in the derivation of this strategy was the knowledge of missile's guidance and target's evading maneuver. For simulation purpose we assume that target performs a constant evasive maneuver of $1.5g$. Missile is assumed to employ proportional navigation with effective gain $N' = 3$.

In order to implement this guidance strategy we need to calculate the relevant elements of the transition matrix by solving the system of differential equations given in Eq. (57). For the current scenario this calculation is similar to the one done in subsection A.

The *Pareto Front* for the current engagement is shown in Fig. 7. The figure represents two optimization objectives of the problem, i.e. missile-defender miss distance and defender control effort. Since the target maneuvers independently, its control effort is constant and equals to $\int u_T^2 dt \simeq 151[m^2/sec]$, no matter what the missile and the defender do. Thus, if we look at the general cooperative pursuit *Pareto Front* from Fig. 2, the *Pareto Front* of the current problem will lay on a plane which corresponds to the cross section at $\int u_T^2 dt \simeq 151[m^2/sec]$ as it is shown in Fig. 8. Similarly to the one-way cooperative defender case the current strategy is an optimal solution under the constraint of a non-cooperative target, and therefore it cannot achieve better results than of a general two-way cooperation unconstrained case. As seen from the Fig. 8 the *Pareto Front* is indeed above the *Pareto Front* surface of a two-way cooperation case. Note that in the current cooperation scheme formulation a scenario with a non maneuvering target (i.e. $u_T = 0$) actually represents a limiting case of the general two-way cooperation with $\beta \to \infty$.

We can observe from Fig. 7 that increasing the penalty α on the miss distance forces the defender to perform harder (its control effort increases) to achieve smaller miss. Further increasing α to infinity ($\alpha \to \infty$) provides us with a limiting case of a perfect interception of the missile by the defender.

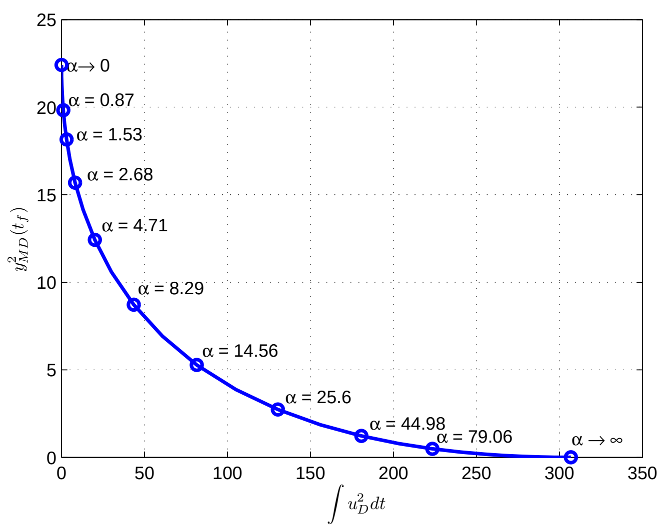

Figure 7. Pareto Front for cooperative defender with independent target.

Evidently, when the defender relies on the ongoing behavior of the target, perfect interception is achievable with a much larger effort than when they both communicate and share their strategies with each other. However, in the previous cooperation scheme where the target used the information on defender's guidance law to support its pursuit a much larger control effort was required from the target. Practically it can be concluded that aiding a defender employing a preset guidance law with a support maneuver of the target is harder since when the future evasion strategy of the target is known the defender just needs to head towards a predicted intercept point.

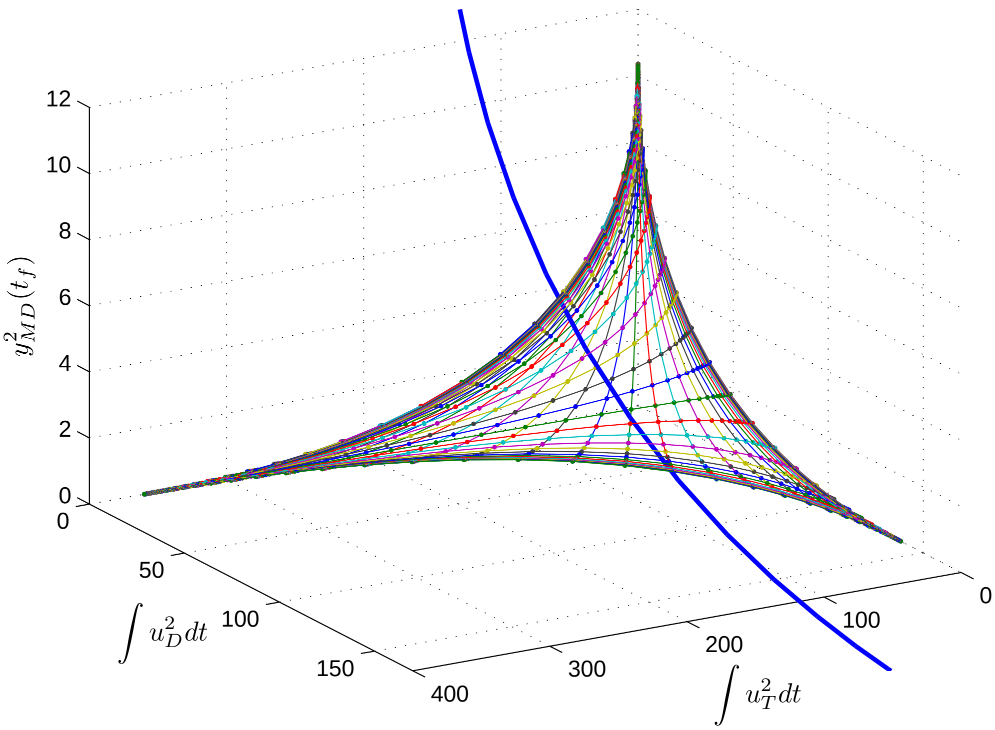

Figure 8. Pareto Front for cooperative defender with independent target compared to full cooperation case.

The navigation gain N'_{D_P} is plotted in Fig. 9 for various values of α including the limiting case where $\alpha \to \infty$. The resulting optimal trajectories are shown in Fig. 10. It is evident that when the objective of the defender is perfect interception it manages to exploit the information about target's maneuver and anticipate the behavior of the missile to null the miss distance.

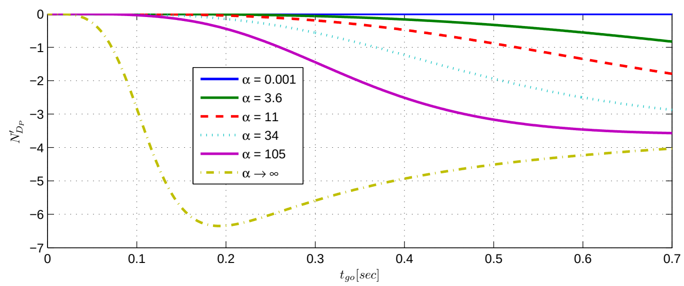

Figure 9. Navigation gain for cooperative defender with independent target.

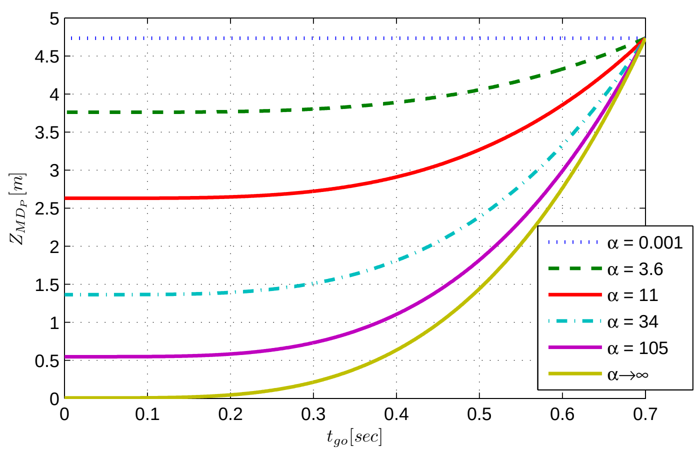

Figure 10. Z_{MD_P} evolution during the engagement.

VII. Conclusions

In this report, that summarizes the work performed in the last stage of the three years research effort, we analyzed an interception engagement in which a defending missile is fired from an aircraft to intercept an incoming homing missile. Three different cooperation schemes were presented and mathematically formulated. For each case optimal cooperative guidance laws were derived, according to the constraints induced by cooperation limitations. The first case implied two-way cooperation allowing full synergy between the target and the defender. In this case no constraints were applied on their behavior which allowed us to derive cooperative pursuit strategies for the target and the defender. In the second case we assumed that only one-way cooperation is available from the side of the target. As a realization of such a scenario we considered an independently homing defender and a target trying to lure in the missile. For this case the optimal one-way cooperative support strategy was derived for the target to aid the defender intercept the missile. Third approach assumed information sharing from the target to the defender, i.e. independently evading target, while the objective of the defender was to exploit this information to intercept the missile. All three guidance schemes were derived assuming arbitrary order linear dynamics of the adversaries, perfect information under the constraints of respective information sharing schemes, and a missile employing a known linear guidance strategy.

Performance of the proposed guidance laws was analyzed via simulation, using the notion of Pareto fronts. As expected, it was shown that fully cooperative actions yield best performance compared to one-way cooperation schemes and one-on-one strategies. Once two-way communication cannot be established and full cooperation is not possible one player of the target-defender team must act independently. It was shown that in such a case it is still possible to intercept a homing missile using an appropriate cooperation scheme. It was also shown that different one-way cooperation schemes impose different maneuvering requirements on the respective cooperatively acting players. Results have shown that cooperatively acting defender is more effective than cooperatively acting target, since cooperative defender only needs to turn towards predicted intercept point, while cooperative target has to take into account missile reaction to its maneuver to bring the independent defender to interception point.

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